2007/50

■

Environmental innovation under Cournot competition

Maria Eugenia Sanin and Skerdilajda Zanaj

CORE DISCUSSION PAPER
2007/50

Environmental innovation under Cournot competition

Maria Eugenia SANIN [1] and Skerdilajda ZANAJ[2]

July 2007

Abstract

In this paper, we address the incentives to invest in environmental innovation of enterprises that exercise market power in the output market and also buy and sell pollution permits. Differently from the existing literature, using a market approach we explicitly model the interaction between the output market, where firms play *à la* Cournot, and the permits market. We find that, in the new equilibrium firms behave symmetrically, that is, they either both innovate to protect their market share in the output market or they both choose not to innovate. Whether the innovation equilibrium arises or not depends on the output demand and on the productivity enhancement and not on the distribution of permits among firms. Finally, we show that, under this market configuration, collusion can be welfare enhancing.

Keywords: environmental innovation, tradable permits, interaction *à la* Cournot.

JEL Classification: D43, L13, Q55

---

[1] CORE and Chair Lhoist Berghmans in Environmental Economics and Management, Université catholique de Louvain, Belgium. E-mail: sanin@core.ucl.ac.be

[2] CORE, Université catholique de Louvain, Belgium and Universita di Siena, Italy. E-mail: skerdilajda.zanaj@uclouvain.be

We wish to thank Thierry Bréchet, Jean J. Gabszewicz, Marco Marinucci and all the attendants to the Environmental Workshops for their stimulating comments.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1 Introduction

In this paper, we address the incentives to invest in environmental innovation of enterprises that exert market power in the output market. These firms also buy and sell pollution permits. Many papers have drawn their attention to the incentives of firms to invest in less pollutant technologies but only Montero (2002) has taken into account strategic interaction in the output market. A major outcome of the previous paper is that incentives to innovate depend on the decrease in permit's price due to investment in R&D[1]. On the contrary, we show that innovation can determine an increase in permits' price, leading to a higher cost of output production. This result is derived using a market approach to model the permits market, rather than a bargaining process.

Moreover, most of the literature regarding market power, claims that the allocation of permits among firms has important efficiency implications. Our main contribution in this regard is that, under Cournot competition, the allocation of permits among firms has no effect neither on efficiency nor on incentives to innovate. In this strategic context, it is shown that innovation incentives mainly depend on output demand and productivity enhancement resulting from technological improvement.

As Montero (2002), most of the papers in this area are devoted to the comparison of the incentives to innovate under alternative pollution control rules. To do so, in general, the interaction between the pollution market and the output market is neglected, or only a single monopoly firm is considered in the output market. This is the case in Wenders (1975), Tietenberg (1985), Downing and White (1986) and Milliman and Prince (1989) who show that market-based instruments such as tradeable permits provide more incentives to environmental innovation than command-and-control instruments. More recently, other authors, pursuing the same objective, introduce explicitly the output market, but most of them consider it as a competitive one (Requate (1995) and Parry (1998)). Most of these papers represents environmental innovation as an R&D sector which produces a (proportional) decrease in the cost of abatement per unit of R&D investment.

[1] In Montero (2002) innovation produces a decrease in permits price that has, on one hand, a direct effect on the innovator profits (positive or negative depending on whether the firms is a buyer or a seller of permits) and, through a decrease in cost of production, allows the innovator's rival to increase output. The incentives to innovate then depend on the net effect and the effect of the decrease in cost of abatement on the investor's profit.

In contrast with this previous literature, we adopt a different definition of environmental innovation. We account for innovation that, instead of simply reducing the marginal cost of abatement[2], produces a change in the intensity of emissions per unit of input used in production, like in Bréchet and Jouvet (2006). This general specification is particularly relevant in the case of some pollutants like C02 where for many sectors reducing emissions requires changing the production process itself[3], and not just investing in some end of pipe technology.

We develop a model of two firms that compete *à la* Cournot in the output market. The decision to innovate is modelled as a sequential game, whose payoffs are composed both from the revenue of the output market and from the revenue or cost from the market for permits. Firms' interaction in the output market gives birth to the demand and the supply of permits in the permits market. Firms receive for free an amount of pollution permits that can be different from the optimal amount of permits needed to produce the output. Accordingly, the "extra" permits of one firm become the supply of permits; and the "shortage" of permits of the other constitutes the demand of permits. In the output market, the price of permits is taken as given, and therefore, the optimal quantities of permits are chosen as a function of their price. This structure reminds the "technology-linked" markets setup proposed by Gabszewicz and Zanaj (2006b). However, there is an important difference between the latter and the present one: in this paper the same firms play both in the output market and in the market for permits. Our innovation game is also reminiscent of the one in Gabszewicz and Garella (1995). The decision to innovate or not is similar to the choice whether or not to internalize production of an intermediate good in the latter paper.

The paper is organized as follows: Section 2 presents the model, the game and the equilibrium analysis, Section 3 discusses welfare efficiency and allocation of permits while Section 4 concludes.

---

[2]This way of modeling environmental innovation is generally inspired on the characteristics of SO2 markets where emissions can be reduced by investing in end of pipe technologies.

[3]Many electricity plants switch to Integrated Gasification Combined Cycle (IGCC) coal generation as they are capable of separating and capturing C02 emissions at a lower cost than conventional coal combustion power plants. For more details on technical issues see Stephens and van der Zwaan (2005).

2 The model

Assume there are two symmetric firms that produce an homogeneous good y using the same technology[4]

$$y = \beta x. \tag{1}$$

Input x is a polluting good. For each firm, emissions e from the use of input x are given by the technology

$$e = \frac{1}{k} x. \tag{2}$$

Hence, technology of producing output y in terms of emissions can be expressed by simply substituting (2) in (1)

$$y = \beta k e. \tag{3}$$

In order to comply with the environmental regulation, each firms must hold one pollution permit for each emission unit. This is what will be call the "full compliance assumption" and allows this paper to talk indistinctly about optimal level of emissions or optimal level of permits used.

Firms face a linear demand for their output, $p = 1 - y$, and they compete *à la* Cournot in the output market. They are assigned for free an amount of permits α and $(1-\alpha)$ respectively, of the total amount of permits S. As long as the amount of permits is assigned equally, there is no exchange of permits. Instead, if firms receive different amount of permits, i.e., $\alpha \neq \frac{1}{2}$, then a market for permits arises. The firm getting a higher number of permits may become the seller of permits, while the firm receiving the smaller share may become the buyer of permits. Our full compliance assumption makes permits a *necessary* input in this polluting industry.

Firms have the possibility to invest in environmental innovation or better to adopt a new technology that is characterized by a lower pollution intensity. Given our interest in markets where the firms emit CO2 to the atmosphere, we characterize the technology change as Hicks-neutral: environmental innovation is a change in the coefficient of emissions' technology

[4]Note that the linear technology that uses only one factor of production is a Cobb-Douglas with a fixed factor.

from k to $\tilde{k}$ where $\tilde{k} > k$; consequently, the marginal productivity of emissions increases[5]. Since we want to highlight the effects of strategic interaction in the innovation decision, we assume that the cost of innovation is the same for both firms, and equal to zero for simplicity. The restrictions $\tilde{k} < 2k$ and $\frac{2}{3} \geqslant \beta S\tilde{k} \geqslant \frac{\tilde{k}-k}{2\tilde{k}-k}$ are imposed throughout the model to guarantee that both firms make non negative profits for all outcomes[6]. Notice that the domain $\{\beta, k, S\}$ satisfying these restrictions is non empty[7].

2.1 The game

Before playing, each firm knows the total amount of permits available S and the proportion α and $(1-\alpha)$ that each of them will get. The innovation game is modelled as a sequential game. Stage one consists of the simultaneous choice whether to invest in innovation or not. In stage two, firms' strategies are output quantities, y^A and y^B respectively, and they are assumed to play Cournot. Accordingly, payoffs in the second stage game obtain as follows:

$$\begin{aligned}\pi^A\left(y^A, y^B\right) &= (1 - y^A - y^B)y^A - q(e^A - \alpha S), \\ \pi^B\left(y^A, y^B\right) &= (1 - y^A - y^B)y^B - q(e^B - (1-\alpha)S).\end{aligned}$$

where q is the price of permits, and e^A and e^B are the amount of emissions emitted by firm A and firm B, respectively.

Denote by (Non innov, Non innov), and call outcome (1) the first-stage choice in which none of the firms chooses to innovate. The payoff pair in this outcome is $\left(\pi_1^A, \pi_1^B\right)$. Similarly, for the other three outcomes of the first-stage game. When firms have to play the first stage game, i.e., the innovation game, under the assumption that their behavior satisfies the criterion of subgame perfection, they consider the matrix game depicted below:

[5] A well-known example where innovation is modelled in the same way can be found in Sharon, Oster (1982).

[6] Firstly, note that the condition $\frac{\tilde{k}}{k} \leq 2$ is necesary for the existence of equilibrium in the permits market. This conditions yields a positively sloped supply of permits coming from the firm receiving $\alpha \geq \frac{1}{2}$ permits. Secondly, equilibrium price of permits is nonnegative if $\beta\tilde{k}S \leqslant \frac{2}{3}$ while equilibrium levels of emissions is nonnegative if $\beta S\tilde{k} \geqslant \frac{\tilde{k}-k}{2\tilde{k}-k}$.

[7] For instance, the set of values $\beta = 0.5, S = 0.5, k = 1$ and $\tilde{k} = 1.3$ satisfies the required system of inequalities.

$$\begin{array}{cccc} & & \multicolumn{2}{c}{\text{B}} \\ & & \textit{Non innov} & \textit{Innovate} \\ \text{A} & \begin{array}{l}\textit{Non innov}\\ \textit{Innovate}\end{array} & \begin{array}{c} 1:\left(\pi_1^A,\pi_1^B\right) \\ 3:\left(\pi_3^A,\pi_3^B\right)\end{array} & \begin{array}{c} 4:\left(\pi_4^A,\pi_4^B\right) \\ 2:\left(\pi_2^A,\pi_2^B\right)\end{array} \end{array} \tag{4}$$

The equilibrium of the game depends on the difference, for each agent, in the payoff coming both from outputs' and permits' markets. The subgame perfect Nash equilibria of the two-stage game are identified through backward induction.

2.2 Second-stage game

2.2.1 Outcome (1): no firm innovates

Output market equilibrium

When no firm innovates, taking into account equation (3), total payoff π^A of firm A obtains

$$\pi^A(e^A, e^B) = (1 - \beta k e^A - \beta k e^B)\beta k e^A - q(e^A - \alpha S). \tag{5}$$

If the amount of permits α assigned to firm A is $\alpha > \frac{1}{2}$, then $\alpha S - e^A$ represents the supply of permits. Similarly, the total payoff of firm B is:

$$\pi^B(e^A, e^B) = (1 - \beta k e^A - \beta k e^B)\beta k e^B - q(e^B - (1 - \alpha)S). \tag{6}$$

Standard computations of Cournot equilibrium give the optimal quantity of emissions chosen by each firm:

$$e_1^A(q) = e_1^B(q) = \frac{\beta k - q}{3\beta^2 k^2}. \tag{7}$$

Permits' market equilibrium

Since firms are symmetric, their optimal level of emissions is the same. Consequently, each firm becomes either a buyer or a seller of permits depending on the amount of permits received (αS or $(1 - \alpha)S$ respectively) as compared with expression (7).

The demand D and supply O of permits are indirectly defined in the optimal decision of the firm in the output market. If $\alpha > \frac{1}{2}$, firm A is the supplier of permits and supply is:

$$O = \alpha S - \frac{\beta k - q}{3\beta^2 k^2}.$$

Analogously, we get the demand of permits as:

$$D = \frac{\beta k - q}{3\beta^2 k^2} - (1-\alpha)S.$$

Then, from the equilibrium condition $O = D$ in the market for permits, the permits price q_1 that clears the market for the entire quantity S obtains as:

$$q_1^* = \beta k(1 - \frac{3}{2}\beta k S). \tag{8}$$

Hence, substituting (8) in (7),

$$e_1^{*A} = \frac{S}{2} = e_1^{*B} \tag{9}$$

Substituting the optimal level of emissions in the production function (3) we find the optimal level of output for each enterprise:

$$y_1^{*A} = \beta k \frac{S}{2} = y_1^{*B} \tag{10}$$

2.2.2 Outcome (2): both firms innovate

Outcome (2) of the second stage game is analogous to outcome (1): firms use the same improved environmental-efficient technology. Production of each firm is now respectively $y^A = \beta\tilde{k}e^A$ and $y^B = \beta\tilde{k}e^B$ where $\tilde{k} > k$. Therefore, for a given amount of emissions, a higher level of production can be obtained.

Following the procedure in the previous section, we find the equilibrium price in the market of permits q_2 when both firms innovate as

$$q_2^* = \beta\tilde{k}(1 - \frac{3}{2}\beta\tilde{k}S). \tag{11}$$

It follows,

Lemma 1 *The permits' price q_2^* when both firms innovate is higher (lower) than the permits price q_1^* when none of the firms innovate if the productivity improvement is lower (higher) than a threshold value.*

Proof. See Appendix. ■

The above lemma claims that, given the exogenous variables $\{\beta, k, S\}$, there is a set of productivity improvement values $\frac{\tilde{k}}{k}$ for which innovation makes the permits price increase. This result is new for the literature concerning pollution permits. The importance of this result is clear as permits' price represent the cost of producing output y having a direct impact on incentives to innovate.

By symmetry, the optimal emissions e_2^A and e_2^B when both firms innovate is equal to

$$e_2^{*A} = \frac{S}{2} = e_2^{*B}. \tag{12}$$

Hence, the optimal output for each enterprise is higher than output in outcome (1) and is equal to

$$y_2^{*A} = \beta\tilde{k}\frac{S}{2} = y_2^{*B}. \tag{13}$$

2.2.3 Outcome (3): only agent A innovates

Output market equilibrium

When only one firm innovates competition is no longer defined as a symmetric Cournot. Instead, the enterprise that innovates is more efficient having a lower unit cost of production. If, in this case, firm A is the one innovating, it maximizes

$$\pi^A(e^A, e^B) = (1 - \beta\tilde{k}e^A - \beta k e^B)\beta\tilde{k}e^A - q(e^A - \alpha S). \tag{14}$$

Similarly, the total payoff of the firm that does not innovate, in this case firm B, is

$$\pi^B(e^A, e^B) = (1 - \beta\tilde{k}e^A - \beta k e^B)\beta k e^B - q(e^B - (1-\alpha)S). \tag{15}$$

Differentiating π^A and π^B, respectively, with respect to e^A and e^B and solving the resulting system, we find the optimal emission for each agent as a function of permits' price, namely,

$$e_3^A(q) = \frac{\left(\tilde{k} - 2k\right) q + \tilde{k}k\beta}{3\tilde{k}^2 k\beta^2} \tag{16}$$

$$e_3^B(q) = \frac{\left(k - 2\tilde{k}\right) q + \tilde{k}k\beta}{3\tilde{k}k^2\beta^2}. \tag{17}$$

Permits' market equilibrium

As before, comparing the optimal level of emissions in (16) and (17) with the amount of permits assigned to each firm, each firm is either a buyer or a seller. This determines the equilibrium price in the market for permits. Therefore from the equilibrium condition[8],

$$\alpha S - \frac{\tilde{k}q - 2kq + \tilde{k}k\beta}{3\tilde{k}^2 k\beta^2} = \frac{kq - 2\tilde{k}q + \tilde{k}k\beta}{3\tilde{k}k^2\beta^2} - (1-\alpha)S$$

the equilibrium permit price is:

$$q_3^* = \frac{1}{2} \frac{\tilde{k}k\beta(\tilde{k} + k - 3k\tilde{k}\beta S)}{\left(\tilde{k}^2 - \tilde{k}k + k^2\right)} \tag{18}$$

Substituting (18) in (16) and (17) we get the optimal level of emissions both for agent A who innovated and for agent B:

$$e_3^{*A} = \frac{\tilde{k} - k - \tilde{k}k\beta S + 2k^2\beta S}{2\beta(\tilde{k}^2 - \tilde{k}k + k^2)} \tag{19}$$

$$e_3^{*B} = \frac{k - \tilde{k} - \beta\tilde{k}kS + 2\beta\tilde{k}^2 S}{2\beta\left(\tilde{k}^2 - \tilde{k}k + k^2\right)} \tag{20}$$

[8] The best response function of agent A, e_3^A, is a function of e_3^B and q; but e_3^B itself is a function of q. Hence, the effect of the price of permits q on e_3^A passes through two channels: a direct one, found in the best response function of agent A, and an indirect one, coming from the effect of q on B's best response function $e_3^A(e_3^B(q))$. If $\tilde{k} > 2k$, the indirect effect overcomes the direct effect, or said differently, the substitution strategic effect intrinsic in the Cournot game overcomes the direct effect of the permits price q. To have a meaningful interaction and the existence of permits market we need the direct effect to be the prelavent one

Then, substituting the optimal level of emissions in the production function we find the optimal level of output for each agent:

$$y_3^{*A} = \tilde{k}\frac{\tilde{k} - k - \beta\tilde{k}kS + 2\beta k^2 S}{2(\tilde{k}^2 - \tilde{k}k + k^2)} \tag{21}$$

$$y_3^{*B} = k\frac{k - \tilde{k} - \beta\tilde{k}kS + 2\beta\tilde{k}^2 S}{2\left(\tilde{k}^2 - \tilde{k}k + k^2\right)} \tag{22}$$

2.2.4 Outcome (4): only agent B innovates

Outcome (4) is analogous to outcome (3) but firm B is the one innovating instead of firm A. This means that, in the output market, firm $B(A)$ will behave in outcome (4) as firm $A(B)$ behaved in outcome (3). Notice that, as total emissions at equilibrium are always exactly equal to S, the fact that only one agent innovates already implies that part of the polluting input is being used more efficiently from an environmental point of view, leading to a higher level of output for the same overall level of emissions S.

Accordingly, taking into account the fact that when both agents simultaneously, innovate, total output increases (see (10) (13)), we can state the following proposition:

Proposition 2 *If at least one firm innovates, total output supplied increases, compared with the output level when no firm innovates.*

Proof. See Appendix. ■

Moreover,

Proposition 3 *The price of permits is independent from the allocation of permits α, in all outcomes of the game.*

Proof. Simple observation of equations (8),(11) and (18). ■

The relevance of permits allocation among agents in terms of price efficiency (i.e. in terms of the monetary burden of pollution control) was studied by Hahn (1984). In this paper, since the dominant firm acts as a monopolist (monopsonist) in order to rise (or depress) permits price, the amount of permits allocated to the dominant determines its position on the permits market and therefore permits price efficiency. In the same line Eshel (2005) finds that α determines the degree of market power of the dominant firm in the permits market and it can "balance" its market power in the output market[9]. Under perfect competition in the output market, the relevance of α is even greater as the amount of permits received is the only opportunity for positive profits.

Instead, we model output market interaction introducing competition *à la* Cournot. This changes the role of permits allocation even in the case of asymmetric Cournot competition: giving to the most efficient agent (or we could say the "dominant " firm in the output market) the exact amount of permits needed for production does not change the price of permits (or unit cost of input)[10]. Then, in our model the value of α has no efficiency implications.

2.3 First-stage game

In this section, firms consider the matrix payoffs (4). Agent A chooses whether to innovate or not for every possible choice of agent B, and vice-versa.

Firm A's best response

Agent A compares the payoff realized in outcome (1) and in outcome (3). Innovating is a best response for A when B is not innovating iff:

$$\Delta\pi^A = \pi_1^A - \pi_3^A < 0. \tag{23}$$

The previous inequality can be developed as:

$$\Delta p y_1^A + \Delta y_1^A p_1 - (\Delta q(e_1^A - \alpha S) + \Delta e q_1) < 0, \tag{24}$$

[9] See Sanin (2007) for further analysis on this issue.

[10] The previous discussion has one further implication: given that efficiency in the permits market does not depend on α, also welfare function becomes independent of α. Hence, in this setup environmental regulation should no longer be concerned with permits allocation among agents.

Moreover, given the social benefit from innovation, the role of the regulator to induce the economy to settle at outcome (2) would be relevant, if possible. But notice that the total amount of permits made available S has no role in the incentives to innovate. This is the case because what makes the economy end up in outcome (2) or (1) is the interaction between output demand and the level of productivity enhancement, both exogenous and given for the regulator.

where $\Delta p = p_1 - p_3$, $\Delta q = q_1 - q_3$, $\Delta e = e_1^A - e_3^A$.

Then, after rearranging, the previous inequality becomes:

$$\frac{\Delta p}{p_1} + \frac{\Delta m^A}{m_1^A} < \frac{1}{\beta k}(\frac{q_1}{p_1}(2 - 2\alpha - \frac{e_3^A}{e_1^A})) + \frac{q_3}{p_1}(2\alpha - 1)) \tag{25}$$

where Δm^A is the change in market share of agent A from outcome (1) to outcome (3), i.e.[11]: $\Delta m^A = m_1^A - m_3^A$.

Firm A compares also the payoff realized in outcome (4) when the other agent B is the only one innovating, with the one he would realize in outcome (2), when both innovate. Innovating is a best response for A when B is innovating iff:

$$\frac{\Delta p}{p_2} + \frac{\Delta m^A}{m_2^A} < \frac{1}{\beta \tilde{k}}(\frac{q_4}{p_2}(2(1-\alpha) - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(1 - 2\alpha)) \tag{26}$$

where $\Delta p = p_4 - p_2$ and $\Delta m^A = m_4^A - m_2^A$.

Firm B's best response

Agent B compares, on one hand, his payoff in outcome (4) versus outcome (1) and, on the other hand, his payoff in outcome (3) versus outcome (2). Given the symmetry of the firms the following conditions, are similar to condition (25) and (26), respectively. In particular, firm B innovates when firm A is not innovating iff:

$$\frac{\Delta p}{p_1} + \frac{\Delta m^B}{m_1^B} < \frac{1}{\beta k}(\frac{q_1}{p_1}(2\alpha - \frac{e_4^B}{e_1^B}) - \frac{q_4}{p_1}(2\alpha - 1)) \tag{27}$$

where $\Delta p = p_1 - p_4$ and $\Delta m^B = m_1^B - m_4^B$.

Instead, firm B innovates when firm A also innovates iff:

$$\frac{\Delta p}{p_2} + \frac{\Delta m^B}{m_2^B} < \frac{1}{\beta \tilde{k}}(\frac{q_3}{p_2}(2\alpha - \frac{e_2^B}{e_3^B}) - \frac{q_2}{p_2}(2\alpha - 1)) \tag{28}$$

[11] Condition (1) imposes a condition for innovation depending on the effect that innovation has on the output market revenue coming *(i)* from a change in output price; *(ii)* on market share$\frac{\Delta p}{p_1} + \frac{\Delta m^A}{m_1^A}$; and *(iii)* the change in the relative price of permits (as these can either be used in the production of y or sold in the permits' market). The RHS of inequality (1) is made of two elements: the first $\frac{\Delta q}{p_1}(1 - 2\alpha)$ is the change in the relative price of the permits as a cost, $\alpha < \frac{1}{2}$, or a unit revenue, $\alpha > \frac{1}{2}$, respectively when the firm is a buyer or a seller of permits. The second component, $\frac{q_1}{p_1}(1 - \frac{e_3^A}{e_1^A})$ is the relative price weighted by the change of permits market position from (1) to (3).

where $\Delta p = p_3 - p_2$ and $\Delta m^B = m_3^B - m_2^B$.

Hence, the pair of strategies (Innovate, Innovate) is a NE equilibrium in the first stage game, if and only if equations (26) and (28) are both satisfied. Notice that, which of these equations is more restrictive for reaching the equilibrium depends, for a given α, on the relationship between prices across outcomes. Then, it follows:

Proposition 4 *If $q_4 < q_2$, outcome (2) INNOVATE-INNOVATE, is a NE of the game iff:*

$$\frac{p_4 - p_2}{p_2} + \frac{m_4^A - m_2^A}{m_2^A} < \frac{1}{\beta \tilde{k}}(\frac{q_4}{p_2}(2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(2\alpha - 1)),$$

while if $q_4 > q_2$,outcome (2) INNOVATE-INNOVATE, is a NE of the game iff:

$$\frac{p_4 - p_2}{p_2} + \frac{m_4^A - m_2^A}{m_2^A} < \frac{1}{\beta \tilde{k}}(\frac{q_4}{p_2}(2 - 2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(1 - 2\alpha)).$$

Proof. See Appendix. ■

Similarly, the pair of strategies (Non Innovate, Non Innovate) is a NE equilibrium in the first stage game if and only if conditions (25) and (27) hold with the reversed sign. Then, depending on to the relationship between $q_1$and q_3 it follows:

Proposition 5 *If $q_1 < q_3$ outcome (1) NOT INNOV-NOT INNOV is an equilibrium of the game iff*

$$\frac{p_1 - p_4}{p_1} + \frac{m_1^B - m_4^B}{m_1^B} > \frac{1}{\beta k}(\frac{q_1}{p_1}(2\alpha - \frac{e_4^B}{e_1^B}) - \frac{q_4}{p_1}(2\alpha - 1)),$$

while if $q_1 > q_3$ outcome (1) is an equilibrium iff

$$\frac{p_1 - p_4}{p_1} + \frac{m_1^B - m_4^B}{m_1^B} > \frac{1}{\beta k}(\frac{q_1}{p_1}(2 - 2\alpha - \frac{e_3^A}{e_1^A}) - \frac{q_3}{p_1}(1 - 2\alpha)).$$

Proof. See Appendix. ■

The uniqueness of the first stage symmetric equilibrium Is it possible that both the pairs of strategies (Innovate, Innovate) and (Non Innovate, Non Innovate) are simultaneously equilibria of the first stage game?

Notice that the sets of parameters $\{\beta, k, S\}$ where multiple equilibria of the game could exist are defined by $q_1 > q_3 = q_4 > q_2$ if $\alpha < \frac{1}{2}$; or $q_1 < q_3 = q_4 < q_2$ if $\alpha > \frac{1}{2}$. The previous question can be answered studying whether the following systems have a solution:

$$\begin{cases} \frac{p_4 - p_2}{p_2} + \frac{m_4^A - m_2^A}{m_2^A} < \frac{1}{\beta \tilde{k}}\left(\frac{q_4}{p_2}(2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(2\alpha - 1)\right) \\ \frac{p_1 - p_4}{p_1} + \frac{m_1^B - m_4^B}{m_1^B} > \frac{1}{\beta k}\left(\frac{q_1}{p_1}(2\alpha - \frac{e_4^B}{e_1^B}) - \frac{q_4}{p_1}(2\alpha - 1)\right) \end{cases}$$

that arise from $q_1 < q_3$ and $q_4 < q_2$, and

$$\begin{cases} \frac{p_4 - p_2}{p_2} + \frac{m_4^A - m_2^A}{m_2^A} < \frac{1}{\beta \tilde{k}}\left(\frac{q_4}{p_2}(2 - 2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(1 - 2\alpha)\right) \\ \frac{p_1 - p_4}{p_1} + \frac{m_1^B - m_4^B}{m_1^B} > \frac{1}{\beta k}\left(\frac{q_1}{p_1}(2 - 2\alpha - \frac{e_3^A}{e_1^A}) - \frac{q_3}{p_1}(1 - 2\alpha)\right) \end{cases}$$

that arise from $q_1 > q_3$ and $q_4 > q_2$.

Proposition 6 *The two pairs of strategies (Innovate, Innovate) and (Not innovate, not innovate) are mutually exclusive.*

Proof. See Appendix. ■

The impossibility of asymmetric equilibria The question is whether it is possible that B prefers outcome (3) to outcome (2) while A prefers outcome (3) to outcome (1) making outcome (3) a possible first stage NE. Given the symmetry between outcome (3) and (4) proving this is the same as proving that outcome (4) is never an equilibrium. The following proposition proves this can never be the case:

Proposition 7 *The first stage game has no asymmetric equilibrium.*

Proof. See Appendix. ■

The previous propositions underlines the importance of the output market characteristics and how such characteristics affect the incentives to innovate through the permits market exchange. In fact, the unit cost for the buyer of permits, q (revenue for the seller), may be higher or lower depending on output demand's characteristics yielding different conditions required for innovation to be the outcome of the game.

Environmental regulation covers different sectors such as energy and transport. Industries like electricity, for example, are often characterized by a well-known inelastic demand (Sanin (2005)) while other sectors like flight transport have a highly elastic demand. This paper emphasizes how industries with different output markets characteristics may differ in their innovation behavior.

3 Welfare considerations

This section is dedicated to study the welfare effect of innovation. The economy is composed of two firms, as described in the above model, and consumers that internalize emissions as a "bad" in their utility function $U(S, y)$.

The equilibrium (Innovation, Innovation) Pareto dominates the equilibrium (Not innovate, Not innovate) if both firms have a higher profit in outcome (2) than outcome (1) with at least one of them with a strictly higher profit. The conditions under which this is the case can be obtained comparing profits of each firm in outcome (1) and in outcome (2), i.e.:

$$\begin{cases} \beta S(\tilde{k}+k) = \frac{4\alpha}{6\alpha-1} \\ \beta S(\tilde{k}+k) < \frac{4(1-\alpha)}{5-6\alpha} \end{cases} \tag{29}$$

or

$$\begin{cases} \beta S(\tilde{k}+k) < \frac{4\alpha}{6\alpha-1} \\ \beta S(\tilde{k}+k) = \frac{4(1-\alpha)}{5-6\alpha} \end{cases} . \tag{30}$$

The investigation of the two systems yields that the innovation outcome Pareto dominates the non innovation outcome if and only if innovation strictly increases each firm' profit. Indeed, innovation is a profitable strategy for each firm if the inequality $\beta S(\tilde{k}+k) < \frac{4\alpha}{6\alpha-1}$ holds for firm A and the inequality $\beta S(\tilde{k}+k) < \frac{4(1-\alpha)}{5-6\alpha}$ holds for firm B. Given α, it is straightforward to check that only one of these condition on $\beta S(\tilde{k}+k)$ is binding, namely the condition bearing on the permits' buyer side[12].

On the other hand, from direct comparison of the sum of profits in outcome (1) and (2), it can be seen that the sum of profits in the innovation

[12] If $\alpha < \frac{1}{2}$, firm A is the buyer of permits and the binding condition is $\beta S(\tilde{k}+k) < \frac{4\alpha}{6\alpha-1}$.While if $\alpha > \frac{1}{2}$, firm B is the buyer and the binding condition is $\beta S(\tilde{k}+k) < \frac{4(1-\alpha)}{5-6\alpha}$.

outcome is higher than the sum of profits in the non-innovation outcome, regardless of productivity enhancement[13]. Moreover, innovation is always welfare improving from the consumers point of view as it implies an increase in total output and a decrease in output price for a given level of pollution. Hence, the question arises on how to induce firms to choose outcome (2) instead of outcome (1) for every play.

One possible answer is allowing for collusion: given that the sum of profits is strictly higher in outcome (2) it could be optimal for the firm that is better off after innovation to pay a compensation to the other firm instead of being in outcome (1). Maximizing collusive profit π_M is as maximizing profits of a single firm that produces total output y. Its profits after innovation are $\pi_M = (1-\beta\tilde{k}e_M)\beta\tilde{k}e_M$ that yield the equilibrium level of emissions $e_M^* = \frac{1}{2\beta\tilde{k}}$. When the cap over emissions is binding, i.e. $\frac{1}{2\beta\tilde{k}} \geq S$, the monopolist uses all permits available S which implies that its price of output is $p_M^* = (1-\beta\tilde{k}S)$, as in the innovation outcome of the Cournot game. Then, collusion is welfare enhancing if receiving half of the collusion profits is higher than the profits that both, the buyer and the seller of permits receive in the non-innovation outcome separately, namely,

$$\left\{ \begin{array}{l} \frac{(1-\beta\tilde{k}S)\beta\tilde{k}S}{2} > \left[(1-\beta kS)\beta k\frac{S}{2} - \beta k(1-\frac{3}{2}\beta kS)(\frac{S}{2}-\alpha S)\right] \\ \frac{(1-\beta\tilde{k}S)\beta\tilde{k}S}{2} > \left[(1-\beta kS)\beta k\frac{S}{2} - \beta k(1-\frac{3}{2}\beta kS)(\frac{S}{2}-(1-\alpha)S)\right] \end{array} \right. , \tag{31}$$

Consider one of the possible combination of parameters that yield a non-innovation equilibrium, i.e. $S = 1, \beta = 0.1, \tilde{k} = 0.4, k = 0.38$, and note that these satisfy $e_M^* = \frac{1}{2\beta\tilde{k}} \geqslant S$. For this combination of parameters both inequalities in (31) are satisfied when α is such that $0.53 > \alpha > 0.47$. This means that, for each combination of parameters that yield a non innovation equilibrium there exists some level of α for which there is place for profitable compensation between firms to achieve the innovation outcome improving welfare. More precisely, when the seller (buyer) does not have a strong long (short) position, each of them is better off agreeing on innovating and dividing collusion profits in halves than achieving the non-innovation outcome through Cournot competition. Therefore, allowing for collusion is welfare improving

[13] The sum of profits in (2) is higher than the sum of profits in (1) if $\frac{p_1}{p_2} < \frac{\tilde{k}}{k}$ that is if $\beta\tilde{k}S > \frac{\tilde{k}}{k+\tilde{k}}$.Recall that the condition $\frac{\tilde{k}-k}{2\tilde{k}-k} < \beta\tilde{k}S$ is always satisfied for feasibility of this model. This latter inequality is more restricting than $\beta\tilde{k}S > \frac{\tilde{k}}{k+\tilde{k}}$.

as firms are better off without harming consumers[14]. It follows,

Proposition 8 *When two Cournot players are also buyer or seller of the input used to produce the output, there exists collusive equilibria that are welfare improving*

Finally, notice that there can be other equilibria where welfare improvement from collusion is even stronger depending on the utility function of consumers. This is the case when the cap on emissions is not binding for the monopolist, i.e. $e_M^* = \frac{1}{2\beta\tilde{k}} < S$ leading to a higher level of environmental quality. Let us assume a separable utility function of consumers where the partial derivative with respect to S is bigger than the partial derivative with respect to consumption of y. In this case there is a larger number of possible welfare improving collusive equilibria.

4 Concluding Remarks

In this paper, we study the incentives for firms to invest in environmental innovation. The decision to innovate is modelled as a sequential game of two firms that compete *à la* Cournot in the output market. Payoffs are composed both from the revenue of the output market and from the revenue or cost from the permits market. In fact, differently from the existing literature, we introduce the innovation game in a framework where both the output market and the permits market are considered. Through the intra-market interaction we establish a link between output demand, productivity enhancement due to innovation, market structure and incentives to innovate.

The main result is that if two symmetric firms compete both in the output and in the permits market, the new equilibrium is again symmetric; whether in this new equilibrium firms innovate or not, depends mainly on the effects that innovation has on the cost of production and therefore on market power in the output market (and on permits market revenue). Previous literature find that when innovation is not costly it is always undertaken as it provokes a decrease in the unit cost of production (price of permits). On the contrary,

[14] Recall that when $\frac{1}{2\beta\tilde{k}} \geq S$, the monopolist uses all permits available S and that price of output is $p_M^* = (1 - \beta\tilde{k}S)$, as in the innovation outcome of the Cournot game. Therefore, consumers pay the same price of output under monopoly or under Cournot competition as well as the same level of pollution S.

we show that innovation can determine an increase in permits' price, leading to a higher cost of output production.

Moreover, most of the literature regarding market power claims that the allocation of permits among firms has important efficiency implications. Our main contribution in this regard is that, under Cournot competition, the allocation of permits among firms has no effect neither on efficiency nor on incentives to innovate. In this strategic context, it is shown that innovation incentives mainly depend on output demand and productivity enhancement resulting from technological improvement.

From the policy makers view point, our main contribution is that the allocation of permits among firms has no effect neither on efficiency nor on incentives to innovate. In this sense we also show that, collusion may be welfare improving.

The paper can be extended in several ways, for a start, end of pipe abatement costs could be introduced to account for abatement's cost reducing innovation. Moreover, dynamics could be introduced in the decisions to innovate taking into account the interaction with the decision to bank permits by the firms. Finally, the model in this paper could also be used to study the incentives of incumbents to deter entry of new arrivals in the output market through actions in the permits market.

5 References

1. T. Bréchet and P-A Jouvet, "Environmental innovation and the cost of pollution abatement", CORE Discussion Paper 40/2006 (2006).

2. P. B. Downing and L. W. White, "Innovation in pollution control", J. Environ. Econom. Management 13, 18–29 (1986).

3. D. M. D. Eshel, "Optimal Allocation of Tradable Pollution Rights and Market Structures", Journal of Regulatory Economics, 28:2 205-223 (2005).

4. J.J. Gabszewicz and P. Garella, "Buy it or make it yourself? A paradox", CORE Discussion paper 14/1995.

5. J. J. Gabszewicz and S. Zanaj, "Upstream Market Foreclosure", CORE DP 2006/97, forthcoming in the Bulletin of Economic Research.

6. J. J. Gabszewicz and S. Zanaj, "Competition in successive markets: entry and mergers",CORE Discussion Paper 97/2006 (2006b).

7. G. O. Gaudet and S. W. Salant, "Uniqueness of Cournot equilibrium: New results from old methods", Rev. Econom. Stud. 58, 399–404 (1991).

8. R.W. Hahn, "Market power and transferable property rights", Quart. J. Econom. 99, 753–765 (1985).

9. S. R. Milliman and R. Prince, "Firms incentives to promote technological change in pollution control", J. Environ. Econom. Management 17, 247–265 (1989).

10. W. S. Misiolek and H. A. Elder, "Exclusionary Manipulation of Markets for Pollution Rights", J. Environ. Econom. Management 16, 156–166 (1988).

11. J.-P. Montero, "Market Structure and Environmental Innovation", Working paper 6-00, Department of Industrial Engineering, Catholic University of Chile (2000).

12. J.-P. Montero, "Permits, Standards, and Technology Innovation", J. Environ. Econom. Management, 44, 23–44 (2002).

13. Sh. Oster, "The Diffusion of Innovation among Steel Firms: The Basic Oxygen Furnace", The Bell Journal of Economics, Vol 13, No 1, pp 45-56.

14. I. Parry, "Pollution regulation and the efficiency gains from technology innovation", J. Regul. Econom. 14, 229–254 (1998).

15. T. Requate, "Incentives to innovate under emission taxes and tradeable permits", European J. Polit. Econom. 14, 139–165 (1998).

16. M. E. Sanin, "On Market Design in Wholesale Electricity Markets", CORE Discussion Paper 100/2006 (2006).

17. M. E. Sanin, "A note on Market Power in Emission Tradable Permits", CORE Discussion Paper forthcoming (2007).

18. T. H. Tietenberg, "Emissions Trading: An Exercise in Reforming Pollution Policy", Resources for the Future, Washington, DC (1985).

19. J. T. Wenders, "Methods of pollution control and the rate of change in pollution abatement technology", Water Resour. Res. 11, 393–396 (1975).

6 Appendix

6.1 Proof of Lemma 1

Substituting $\tilde{k}$ in (11) by ak with $a \in (1,2)$ representing the productivity enhacement.

Then, the derivative of (11) with respect to a is:
$\frac{\partial q_2^*}{\partial a} = \frac{\partial \beta ak(1-\frac{3}{2}\beta akS)}{\partial a} = \beta k(1-3\beta kSa)$ that can be possitive or negative. Particularly, it is possitive for a set of values of $\{\beta, k, S\}$ for which $a < \frac{1}{3\beta kS}$.

6.2 Proof of Proposition 2

Since it is the case that $e^A = S - e^B$, the total output in outcome (3) can be written as

$y^A + y^B = \beta\tilde{k}S - \beta\tilde{k}e^B + \beta ke^B = \beta\tilde{k}S - \beta e^B(\tilde{k} - k)$.

Output in outcome (1) instead is βkS.

Then, output in (3) is higher than in (1) if $\beta\tilde{k}S - \beta e^B(\tilde{k} - k) > \beta kS$ that after rearranging becomes $S > e^B$ which is always true.

6.3 Proof of Proposition 3

Step 1: Recall condition (26) is

$$\frac{\Delta p}{p_2} + \frac{\Delta m^A}{m_2^A} < \frac{1}{\beta\tilde{k}}(\frac{q_4}{p_2}(2(1-\alpha) - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(1-2\alpha))$$

and that in equilibrium it is the case that: $p_3 = p_4; q_3 = q_4; e_2^A = e_2^B; m_3^B = m_4^A; m_2^B = m_2^A$.

Then, substituting the previous inequalities in condition **(28)** for B, this condition becomes:

$$\frac{\Delta p}{p_2} + \frac{\Delta m^A}{m_2^A} < \frac{1}{\beta\tilde{k}}(\frac{q_4}{p_2}(2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(2\alpha - 1)).$$

Then, condition (28) is more binding than condition in (26) if

$$\frac{1}{\beta\tilde{k}}(\frac{q_4}{p_2}(2 - 2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(1-2\alpha)) > \frac{1}{\beta\tilde{k}}(\frac{q_4}{p_2}(2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(2\alpha - 1))$$

that after rearranging becomes a condition on the relationship among the equilibrium price of permits and α:

$$q_4(\frac{1}{2}-\alpha) > q_2(\frac{1}{2}-\alpha)$$

If $\alpha > (<)\frac{1}{2}$ condition (28) is binding if $q_4 < (>)q_2$, otherwise condition in (26) is binding.

Step 2:Then, for a given α, wether q_4 is higher or lower than q_2 determines wich condition needs to be satisfied for having an INNOVATION-INNOVATION equilibrium.

6.4 Proof of Proposition 4

Step 1: Recall that condition (25) is not satified when

$$\frac{\Delta p}{p_1} + \frac{\Delta m_1^A}{m_1^A} > \frac{1}{\beta k}(\frac{q_1}{p_1}(2(1-\alpha) - \frac{e_3^A}{e_1^A}) - \frac{q_3}{p_1}(1-2\alpha))$$

and that in equilibrium it is the case that $p_3 = p_4; q_3 = q_4; e_2^A = e_2^B; m_3^B = m_4^A; m_2^B = m_2^A$.

Substituting the previous values in the condition of violation of (27) we find that the condition under wich it is optimal for B not to innovate is more binding that the condition for wich it is optimal for A not innovate (violation of (25)) if:

$$\frac{1}{\beta k}(\frac{q_1}{p_1}(2\alpha - \frac{e_4^B}{e_1^B}) - \frac{q_4}{p_1}(2\alpha-1)) > \frac{1}{\beta k}(\frac{q_1}{p_1}(2(1-\alpha) - \frac{e_3^A}{e_1^A}) - \frac{q_3}{p_1}(1-2\alpha))$$

that after rearranging becomes a condition on the relationship among the equilibrium price of permits and α:

$$q_4(\alpha - \frac{1}{2}) > q_1(\alpha - \frac{1}{2})$$

If $\alpha > (<)\frac{1}{2}$, violation of condition (27) is binding if $q_4 > (<)q_1$, otherwise violation of condition (25) is binding.

Step 2:Then, for a given α, wether q_4 is higher or lower than q_1 determines wich condition needs to be satisfied for having an NON INNOVATION-NON INNOVATION equilibrium.

6.5 Proof of Proposition 5: Uniqueness

Step 1: Consider the system between the condition for the innovation equilibrium to arrise and the condition for the non-innovation equilibrium:

$$\begin{cases} \frac{p_4-p_2}{p_2}+\frac{m_4^A-m_2^A}{m_2^A}<\frac{1}{\beta\tilde{k}}(\frac{q_4}{p_2}(2\alpha-\frac{e_2^A}{e_4^A})-\frac{q_2}{p_2}(2\alpha-1)) \\ \frac{1}{\beta k}(\frac{q_1}{p_1}(2\alpha-\frac{e_4^B}{e_1^B})-\frac{q_4}{p_1}(2\alpha-1))<\frac{p_1-p_4}{p_1}+\frac{m_1^B-m_4^B}{m_1^B} \end{cases}$$

Step 2: We prove that both inequalities cannot be satisfied simultaneously.

We know that: $p_3=p_4; q_3=q_4; e_2^A=e_2^B; m_3^B=m_4^A; m_2^B=m_2^A$

The first inequality can be simplified as:

$$2\beta\tilde{k}(m_4^A-\frac{1}{2})<(2\alpha-\beta\tilde{k})\frac{q_4-q_2}{p_2}-\frac{q_4}{p_2}\frac{S}{2e_4^A}+\frac{q_2}{p_2}$$

where $m_4^A=(1-m_4^B)$ and $e_4^B=S-e_4^A$ and therefore

$$2\beta\tilde{k}(\frac{1}{2}-m_4^B)<(2\alpha-\beta\tilde{k})\frac{q_4-q_2}{p_2}-\frac{q_4}{p_2}\frac{S}{2e_4^A}+\frac{q_2}{p_2} \tag{32}$$

Similarly, the second inequality can be rewriten as:

$$(2\alpha-\beta k)\frac{q_1-q_4}{p_1}-\frac{q_1}{p_1}(2-\frac{2e_4^A}{S})+\frac{q_4}{p_1}<2\beta k(\frac{1}{2}-m_4^B) \tag{33}$$

As it is always the case that $k<\tilde{k}$, the LHS of (32) is higher than RHS of (33).

Therefore, there is no multiple equilibria if the following inequality is violated:

$$\left((2\alpha-\beta k)\left(q_1-q_4\right)-q_1(2-\tfrac{2e_4^A}{S})+q_4\right)< \frac{p_1}{p_2}\left((2\alpha-\beta\tilde{k})\left(q_4-q_2\right)-q_4\tfrac{S}{2e_4^A}+q_2\right)$$

Substituting inside the previous $q_4=\frac{1}{2}\frac{\tilde{k}k\beta(\tilde{k}+k-3k\tilde{k}\beta S)}{\left(\tilde{k}^2-\tilde{k}k+k^2\right)}$; $e_4^A=\frac{k-\tilde{k}-\beta\tilde{k}kS+2\beta\tilde{k}^2S}{2\beta\left(\tilde{k}^2-\tilde{k}k+k^2\right)}$; $q_2=\beta\tilde{k}(1-\frac{3}{2}\beta\tilde{k}S); q_1=\beta k(1-\frac{3}{2}\beta kS)$

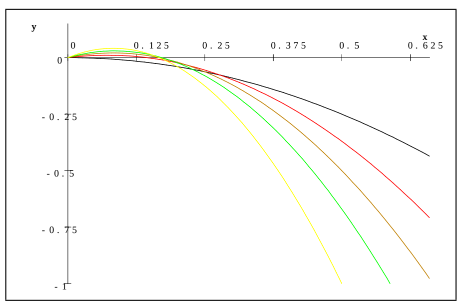
and taking into account that $p=1-y$ we show that proving that the previous inequality is violated simply means proving the following positive:

$$\frac{1}{6}\frac{\left(k-\tilde{k}\right)^2\left(\begin{array}{c}6k-21\tilde{k}k-2\tilde{k}^2+13\tilde{k}^3-27\tilde{k}^4+\\ 18\tilde{k}^5-13k^2\\ +9k^4+43\tilde{k}k^2+29\tilde{k}^2k-3\tilde{k}k^3-\\ 30\tilde{k}^3k-18\tilde{k}k^4+36\tilde{k}^4k\\ -18\tilde{k}^5k-60\tilde{k}^2k^2+18\tilde{k}^2k^3+36\tilde{k}^3k^2\\ +9\tilde{k}^2k^4-9\tilde{k}^3k^3-9\tilde{k}^4k^2\end{array}\right)}{\left(\tilde{k}-k+\tilde{k}k-2\tilde{k}^2\right)\left(\tilde{k}^2-\tilde{k}k+k^2\right)\left(\tilde{k}-1\right)}>0 \tag{34}$$

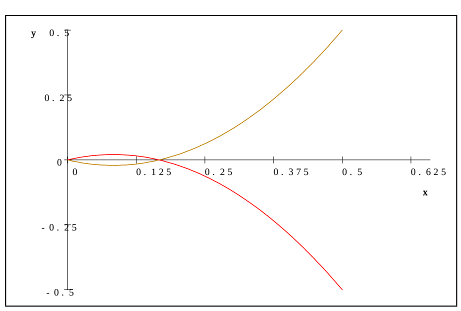
Now we study the sign of each of the functions in (34):
1.

$$\tilde{k}-k+\tilde{k}k-2\tilde{k}^2<0$$

Lets call $\tilde{k}=ak$. For different values of $a\in(1,2)$ the previous function is positive or negative depending on the value of k (see graph):

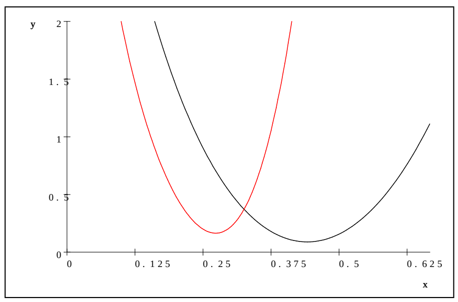


But in the admissible set of parameters defined by the condition on positivity of emmisions $\beta S\tilde{k}\geqslant\frac{\tilde{k}-k}{2\tilde{k}-k}$ the previous function is always negative. When the denominator of emmissions of B in outcome (3) (or emissions of A in outcome (4)) and the previous function $\tilde{k}-k+\tilde{k}k-2\tilde{k}^2$) is positive the function $\tilde{k}-k+\tilde{k}k-2\tilde{k}^2$ is negative (see graph):



2.
$$\left(\begin{array}{c} 6k-21\tilde{k}k-2\tilde{k}^2+13\tilde{k}^3-27\tilde{k}^4 \\ +18\tilde{k}^5-13k^2+9k^4+43\tilde{k}k^2 \\ +29\tilde{k}^2k-3\tilde{k}k^3-30\tilde{k}^3k-18\tilde{k}k^4 \\ +36\tilde{k}^4k-18\tilde{k}^5k- \\ 60\tilde{k}^2k^2+18\tilde{k}^2k^3+36\tilde{k}^3k^2 \\ +9\tilde{k}^2k^4-9\tilde{k}^3k^3-9\tilde{k}^4k^2 \end{array}\right)>0$$

Lets call again $\tilde{k}=ak$. For different values of $a\in(1,2)$ the previous function is always possitive in the admissible set of parameters (see graph):



3.
$$\frac{1}{6}\frac{\left(k-\tilde{k}\right)^2}{\left(\tilde{k}^2-\tilde{k}k+k^2\right)\left(\tilde{k}-1\right)}<0 \tag{35}$$

As $\tilde{k}^2>\tilde{k}k$ this function is negative due to the negative component $(\tilde{k}-1)$. It is the case that $\tilde{k}<1$ to ensure that output price $p_2=1-\beta\tilde{k}S$ is possitive as for this proof we take $\beta=1, S=1$.

Hence, (34) is satisfied meaning we have a unique equilibriun in the admissible set of $\beta, k, \tilde{k}, S$.

Step 3: The incompatibility of the following system can be proven following the previous reasoning:
$$\begin{cases} \frac{p_4-p_2}{p_2}+\frac{m_4^A-m_2^A}{m_2^A}<\frac{1}{\beta\tilde{k}}\left(\frac{q_4}{p_2}\left(2-2\alpha-\frac{e_2^A}{e_4^A}\right)-\frac{q_2}{p_2}(1-2\alpha)\right) \\ \frac{p_1-p_4}{p_1}+\frac{m_1^B-m_4^B}{m_1^B}>\frac{1}{\beta k}\left(\frac{q_1}{p_1}\left(2-2\alpha-\frac{e_3^A}{e_1^A}\right)-\frac{q_3}{p_1}(1-2\alpha)\right) \end{cases}$$

6.6 Proof of Proposition 6: Imposibility of asymmetric equilibria

Could it be the case that, at the same time preference between possible outcomes of the game satisfy[15] $(3) \succ_B (2)$ and $(3) \succ_A (1)$?

Recall condition (25) that ensures $(3) \succ_A (1)$:

$$\frac{p_1 - p_3}{p_1} + \frac{m_1^A - m_3^A}{m_1^A} < \frac{1}{\beta k}\left(\frac{q_1}{p_1}(2(1-\alpha) - \frac{e_3^A}{e_1^A}) - \frac{q_3}{p_1}(1-2\alpha)\right)$$

While the condition that ensures $(3) \succ_B (2)$ is condition (28) with the reverse sign:

$$\frac{p_4 - p_2}{p_2} + \frac{m_4^A - m_2^A}{m_2^A} > \frac{1}{\beta \tilde{k}}\left(\frac{q_4}{p_2}(2\alpha - \frac{e_2^A}{e_4^A}) - \frac{q_2}{p_2}(2\alpha - 1)\right)$$

If the previous two inequalities are satisfied simultaneously there exists an asymetric equilibria. Then, after rearranging the system becomes:

$$\begin{cases} 2\beta k(1 - m_3^A) < \frac{1}{p_1}\left(q_1(2(1-\alpha) - \frac{e_3^A}{e_1^A}) - q_3(1-2\alpha) + p_3\beta k\right) \\ 2\beta \tilde{k} m_3^A < \frac{1}{p_2}\left(q_3(\frac{e_1^A}{e_3^B} - 2\alpha) + q_2(2\alpha - 1) + p_3\beta\tilde{k}\right) \end{cases}$$

where $m_3^A > \frac{1}{2}$ and $k < \tilde{k}$.

Then, the LHS of the first inequality in the system is lower than LHS of the second inequality meaning that they are satisfied simultaneously if:

$$2\beta k m_3^B < 2\beta\tilde{k} m_3^A < \frac{1}{p_2}\left(q_3(\frac{e_1^A}{e_3^B} - 2\alpha) + q_2(2\alpha - 1) + p_3\beta\tilde{k}\right)$$

Then, for finding the set of parameters for which an assymetric equilibria is not possible we must find the parameters for wich the following inequality is true:

$$2\beta k m_3^B - \frac{1}{p_2}\left(q_3(\frac{\frac{S}{2}}{e_3^B} - 2\alpha) + q_2(2\alpha - 1) + p_3\beta\tilde{k}\right) > 0 \tag{36}$$

Substituting equilibrium values: $q_3 = \frac{1}{2}\frac{\tilde{k}k\beta(\tilde{k}+k-3k\tilde{k}\beta S)}{\left(\tilde{k}^2 - \tilde{k}k + k^2\right)}$; $q_2 = \beta\tilde{k}(1 - \frac{3}{2}\beta\tilde{k}S)$; $e_3^B = \frac{k - \tilde{k} - \beta\tilde{k}kS + 2\beta\tilde{k}^2 S}{2\beta\left(\tilde{k}^2 - \tilde{k}k + k^2\right)}$

[15] By symetry, this question is the same as asking both $(4) \succ_A (2))$ and $(4) \succ_B (1))$ to be satisfied simultaneously.

and valuating in admissible values condition (36) becomes:

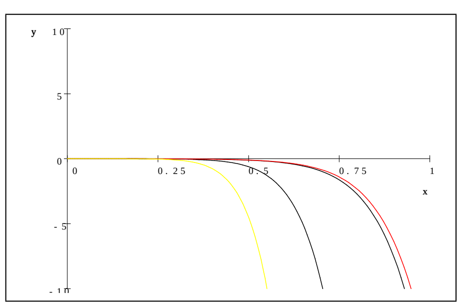
$$\left(-\frac{1}{6}\right)\frac{\left(k-\tilde{k}\right)\left(\begin{array}{c}\tilde{k}^5+\tilde{k}^6-6\tilde{k}^7-12k^5+23\tilde{k}k^4- \\ 2\tilde{k}^4k+39\tilde{k}k^5-11\tilde{k}^5k+22\tilde{k}^6k- \\ 6\tilde{k}^7k-22\tilde{k}^2k^3+12\tilde{k}^3k^2-93\tilde{k}^2k^4 \\ +74\tilde{k}^3k^3-34\tilde{k}^4k^2-42\tilde{k}^2k^5 \\ +145\tilde{k}^3k^4-120\tilde{k}^4k^3+ \\ 63\tilde{k}^5k^2+3\tilde{k}^3k^5-63\tilde{k}^4k^4 \\ +57\tilde{k}^5k^3-45\tilde{k}^6k^2\end{array}\right)}{\begin{array}{c}\left(\tilde{k}^2-2\tilde{k}k+k^2+\tilde{k}k^2+\tilde{k}^2k\right)\left(\tilde{k}^2-\tilde{k}k+k^2\right) \\ \left(\tilde{k}-k+\tilde{k}k-2\tilde{k}^2\right)\left(\tilde{k}-1\right)\end{array}}>0 \qquad (37)$$

Now we study the sign of the functions in (37):

1. Given that we assumed $\beta=S=1$, $(\tilde{k}-1)<0$
2. $-\frac{1}{6}<0$
3. $\left(\tilde{k}^2-\tilde{k}k+k^2\right)>0$ as $\tilde{k}^2>\tilde{k}k$
4. $\tilde{k}-k+\tilde{k}k-2\tilde{k}^2<0$ already studied in the previous proof: always negative in the admisible set of parameters.
5.

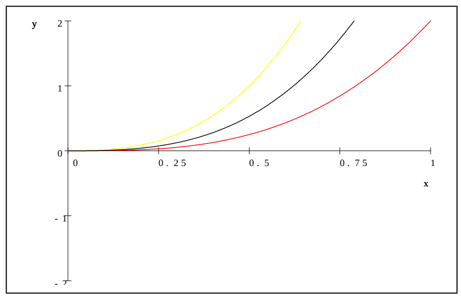
$$\left(\begin{array}{c}\tilde{k}^5+\tilde{k}^6-6\tilde{k}^7-12k^5+23\tilde{k}k^4-2\tilde{k}^4k+39\tilde{k}k^5 \\ -11\tilde{k}^5k+22\tilde{k}^6k-6\tilde{k}^7k-22\tilde{k}^2k^3 \\ +12\tilde{k}^3k^2-93\tilde{k}^2k^4+74\tilde{k}^3k^3-34\tilde{k}^4k^2 \\ -42\tilde{k}^2k^5+145\tilde{k}^3k^4-120\tilde{k}^4k^3+63\tilde{k}^5k^2 \\ +3\tilde{k}^3k^5-63\tilde{k}^4k^4+57\tilde{k}^5k^3-45\tilde{k}^6k^2\end{array}\right)<0$$

Taking $\tilde{k}=ak$, the previous function is always negative for different values of $a\in(1,2)$ (see graph):



6. $\left(\tilde{k}^2 - 2\tilde{k}k + k^2 + \tilde{k}k^2 + \tilde{k}^2 k\right) > 0$

Again taking $\tilde{k} = ak$ the previous function mapped for different values of $a \in (1, 2)$ is always positive (see graph).



Hence, the sign of the overall function is positive meaning that there is no possible asymetric equilibria in the admisible set of $\beta, k, \tilde{k}, S$. Then, we have proved that the symmetric equilibria are unique.